7 HOOKE'S LAW

7.1 Elasticity

Elasticity is the ability of a material to recover its original shape and size after the force deforming it has been removed. For example, a rubber band snaps back to its original size after being stretched as shown in Fig. 7.1 (a). Similarly, a metre rule straightens up when a slight bending force is withdrawn.

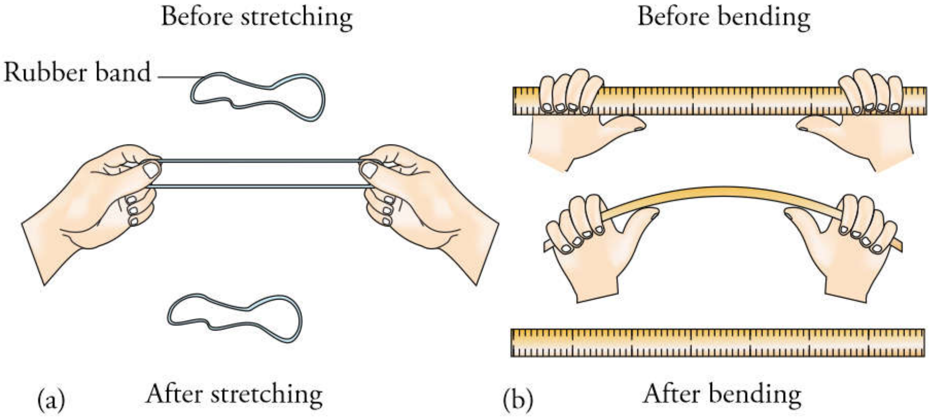


Fig. 7.1: Elasticity of a rubber band and a metre rule

A material is said to be more elastic if it gets back more precisely to its original shape and size. A piano wire, for example, is more elastic than a rubber band. Coil springs, also called helical springs (since their mathematical shape is called helix) are also elastic.

When a spring is stretched by a force, it increases in length; more and more as the force increases. Generally, the applied force is proportional to the amount of extension, as described by Hooke's Law. However, there is a limit to the size of the force beyond the material will permanently deformed. This limit is called elastic limit.

Material that get permanently deformed when a force is applied on them are no-elastic. Examples of non-elastic materials include plasticine, dough and soap.

Activity 7.1: Investigating the relationship between force and extension of a spring

The requirements are; a spring with a pointer, six 50 g masses with hooks, retort stand with clamp and a metre rule.

- Suspend the spiral spring on the stand with the pointer at the bottom
- Clamp the metre rule retort stand close to the spring
- Note and record in a table the initial pointer reading (x_o) on the metre rule. See Fig. 7.2 (a).
- Hook one 50 g mass on the spring, just below the pointer, note and record the new pointer reading (x_1). See Fig. 7.2 (b).
- Determine the extension (e) of the spring from the two values. $e = x_1 - x_o$
- Repeat this with the addition of one 50 g mass at a time
- Plot a graph of force against extension of the spring

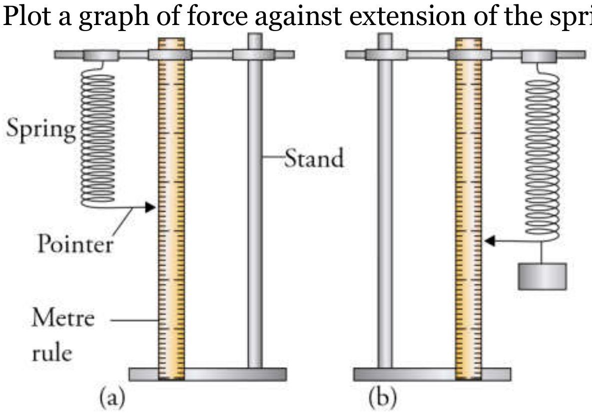


Fig. 7.2: Investigating the relationship between force and extension

Table 7-1: Table of data

Original position of the spring when no mass is attached = x_o			
Mass (g)	**Force (N)**	**Pointer position (cm)**	**Extension (e) (x_1 - x_o) cm**
50	0.5	x_1	
100	1.0	x_2	
150	1.5	x_3	
200	2.0	x_4	
250	2.5	x_5	
300	3.0	x_6	

A graph of force against extension for the above collected data is a straight-line graph through the origin.

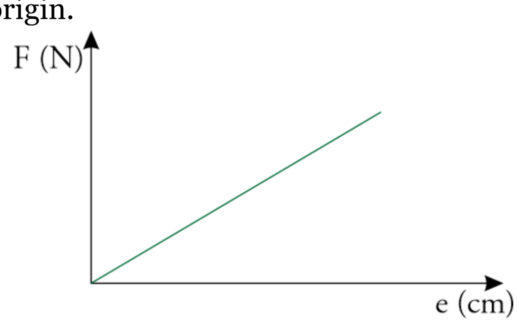


Fig. 7.3: Force against extension graph sketch

A straight line through the origin has a constant gradient and it implies that the components plotted are directly proportional. It can therefore be concluded that the extension of a spring is directly proportional to the force attached. This relationship is described by Hooke's law.

7.2 Hooke's law

It describes the relationship between extension of a stretching material and the stretching force. It states that

The extension of a spring is directly proportional to the applied force, provided that the elastic limit of the spring is not exceeded

This law can be mathematically expressed as;

Force α extension

$F \alpha e$

$\therefore F = ke$

In the equation, k is the constant of proportionality and is called the springs constant. A spring with a large spring constant is stiffer and extends just a little when a force is applied to it. Graphically, Hooke's law can be represented as shown in Fig. 7.4 below.

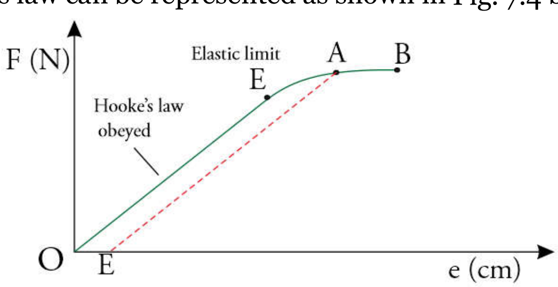


Fig. 7.4

Form the graph;

1. Region OE represents the proportionality part. Within the region, extension is directly proportional to force. Hooke's law is obeyed
2. Point represents the elastic limit of the spring. The value of force at this point is the maximum force for Hooke's law to be obeyed. Any value above this deforms the spring.
3. EAB is a curve where Hooke's law is no longer obeyed. Extension and force are not directly proportional. A force produces a very large extension. A small permanent extension is caused in the spring and the spring in no longer elastic. If the force was to be removed at point A for example, the permanent extension is the length OE.

NOTE: Lines of best fit.
When graphing values from a table, straight lines are obtained by drawing a line of best fit.
The 'line of best fit' is a line that goes **roughly through the middle of all the scatter points** on a graph. The line of best fit is drawn so that the points are evenly distributed on either side of the line.

Example 7.1
The following results were obtained in an experiment to verify Hooke's law when a helical spring was extended by hanging various loads on it.

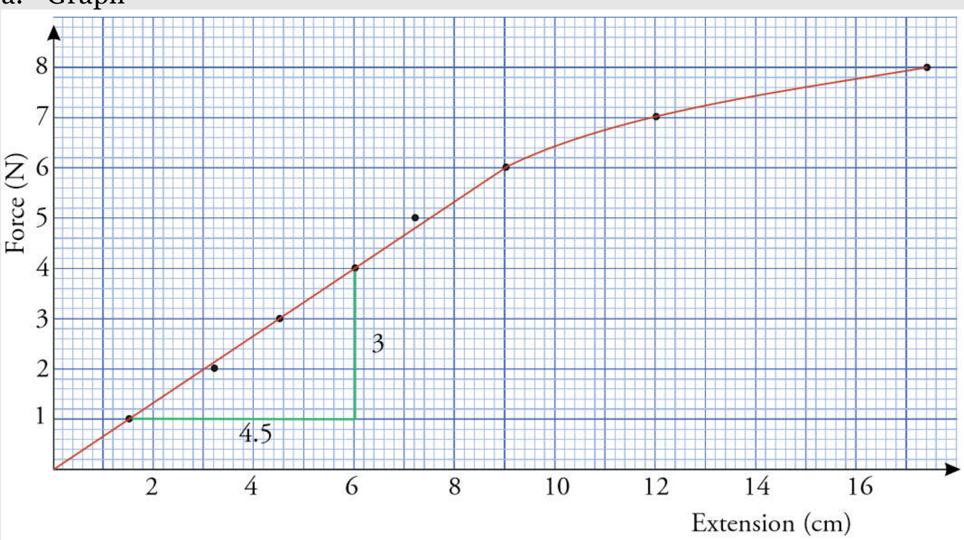
Load, F (N)	0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00	8.00
Length of spring (cm)	8.0	9.5	11.2	12.4	14.1	15.8	17.3	21.0	29.5
Extension, e (cm)	**0.00**								

a. Complete the table for extension, e
b. Plot a graph of Load (y-axis) against extension
c. From the graph, determine;
   i. the spring constant, k
   ii. the elastic limit of the spring

Solution

Load, L(N)	0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00	8.00
Length of spring (cm)	8.0	9.5	11.2	12.6	14.0	15.4	17.0	20.0	29.5
Extension, e (cm)	0.0	1.5	3.2	4.6	6.0	7.4	9.0	12.0	21.5

a. Graph



b. Spring constant, k = slope of graph = $\frac{\Delta Force}{\Delta extension}$

$$k = \frac{4.0 - 1.0}{6.0 - 1.5} = \frac{3}{4.5} = 0.6667\ Ncm^{-1}$$

c. Elastic limit = 6 N

Example 7.2
A mass 1500 g when hung from the lower end of a spring which obeys Hooke's law extends the spring by 16 cm. Determine the spring constant for this spring in SI units.

Solution

$$F = ke$$
$$F = mg = 1.5 \times 10 = 15\ N$$
$$e = 0.25\,m$$
$$k = \frac{F}{e} = \frac{15N}{0.25\,m} = 60\ N/m$$

Example 7.3

A body of mass 560 g causes an extension of 2.8 mm to a certain copper wire. Determine the mass that will cause a 3.6 mm extension on the same wire given that the wire obeys Hooke's law.

Solution

$F = ke$

$k = \frac{F}{e} = \frac{5.6N}{2.8mm} = 2.0N/mm$

$When\ e = 3.6mm$

$F = 3.6 \times 2 = 7.2N$

Example 7.4

A spring obeying Hooke's law was suspended on a fixed support. When a load of 6 N load was hung on it, the length of the spring was 18 cm. When a 30 N load was hung on it, the length increased to 30 cm. Determine:

a. the length of the spring without a load
b. the length of the spring when supporting a load of 20 N.

Solution

a. In this case, a force of 24 N (30 N – 6 N) produces an extension of 12 m (30 cm -18 cm)
Determine the extension produced by the 5 N

$e = \frac{F}{k} = \frac{6}{2} = 3\ cm$

Original length pf the spring = 18 cm – 3 cm =15 cm

b. Determine the extension for 20 N

$e = \frac{F}{k} = \frac{20}{2} = 10\ cm$

Add to the original length
Length=10 + 15 cm = 25 cm

Understanding spring constant

Spring constant can be likened to the stiffness of a spring. A spring that is stiffer has a large spring constant. Consider two cases below:

1. *A force of 5 N extends spring P by 25 cm.*
We determine the spring constant by applying

$F = ke$

$k = \frac{F}{e} = \frac{5N}{0.25\ m}$

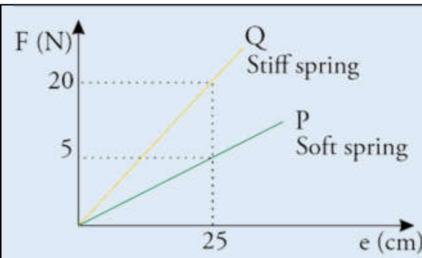
$= 20Nm^{-1}$

2. *A force of 20 N extends spring Q by 25 cm.*

$F = ke$

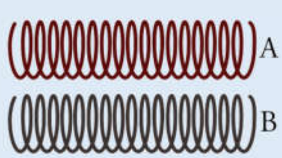
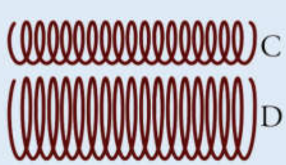
$k = \frac{F}{e} = \frac{20N}{0.25\ m}$

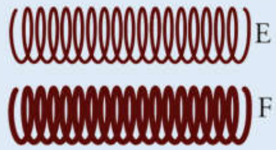
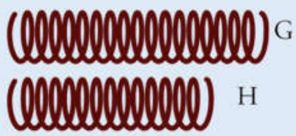
$= 80Nm^{-1}$



With a large value of k, spring Q requires more force to extend by the same value as P. Spring Q is thus stiffer than P.

Factors affecting spring constant of a spring

1. Type of the material 	Identical springs of different materials will have different spring constants. For example, if spring is made of copper and B made from steel, then spring B is stiffer than A
2. Diameter of the spring 	The stiffness (spring constant increases) increases with the decrease in diameter. Spring C is stiffer than spring D.

3. Thickness of the wire 	A spring made of a thicker wire is stiffer than the one made of thin wire of the same material. F has stiffer than E
4. Length of the spring 	A shorter spring is stiffer than a longer one. The spring constant of H is greater than that of G

Test yourself 7.1

1. A force of 60 N pulls a spring from 10 cm mark to 22 cm mark. What is the spring constant of the spring?
2. A spring has a length of 15 cm. How much force is required to pull it through 18 cm if the spring has a spring constant of 100 Nm^{-1}?
3. A form 2 student carried out an experiment to investigate Hooke's law using a spring a number of masses. The results of the experiment are shown in the table.

Force (N)	0	2	4	6	8	10	12	14	16
Extension (cm)	0.0	1.6	3.2	5.0	6.4	7.8	9.6	11.2	12.8

   a. Use these results to plot a graph of force against extension
   b. Determine the spring constant of the spring from the graph
   c. Use the graph to find
      i. the force that produces and extension of 7.0 cm
      ii. the extension when the force is 5 N
4. A certain spring has a length of 40 cm. The spring is placed vertically on a table and a mass of 15 N is placed its top. The spring's length then becomes 32 cm. Find the spring constant.

7.3 Energy stored in a spring

Work is done in stretching or compressing a spring. This work done is equivalent to the energy stored in the spring. The SI unit of work and energy is the Joule (J). The energy is called elastic potential energy and is given by;

$$Work\ done = Energy\ stored = \frac{1}{2}Fe$$

$$W = E = \frac{1}{2}Fe = \frac{1}{2}ke^2$$

In the graph of force against extension, work done is equivalent to the area under the curve.

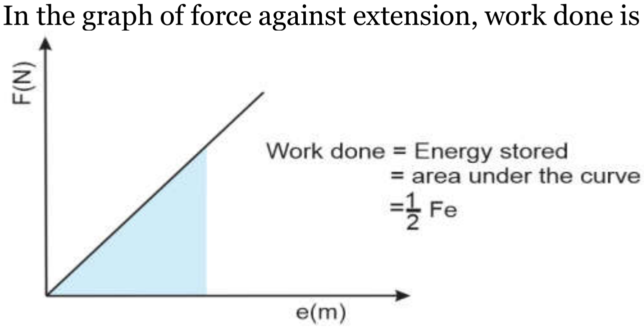


Fig. 7.5: Work done in stretching a spring

Example 7.5

A force of 50 N is used to stretch a certain spring through 8 cm. Calculate the elastic potential energy stored in the spring.

Solution

$$Elastic\ P.E = \frac{1}{2}Fe$$
$$= \frac{1}{2}\times 50\times 0.08 = 2\,J$$

7.4 Compression of a spring

When a spring is squeezed, it reduces in size. The change in length as the spring compresses is called compression. Hooke's law still applies in compression of a spring. The compression is directly proportional to the compressing force.

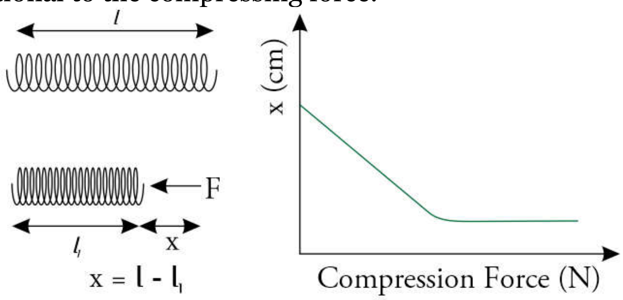


Fig. 7.6: Compression of a spring

Fig. 7.6 above shows change in length when compressing a spring alongside a graph of compression against the compression force.

Example 7.6

A 20 cm spring is compressed by a 200 g mass to 25 cm. Determine the additional mass required to compress the spring to 20 cm

Solution

$F = ke$

$F = 2\ N,\ e = 20 - 15 = 5\ cm$

$k = \frac{F}{e} = \frac{2}{5}\ = 0.4Ncm^{-1}$

$e = 20 - 10 = 10\ cm$

$F = ke$

$F = 0.4\times 10\ = 4N$

$4\ N = 400\ g.$

$Additional\ mass\ =\ 400\ g - 200\ g =\ 200\ g$

Test yourself 7.2

1. A nylon string with original length 20 cm, is pulled by a force of 25 N. The change in length of the string is 5 cm. Determine the magnitude of force if the change in length is 12 cm.
2. A spring has a spring constant of 50 N/m is stretched through an extension of 25 cm. Assuming that it obeys Hooke's law, calculate the elastic potential energy stored by the spring.
3. A force of 500 N is used to compress a certain spring. Given that the potential energy stored in the spring is 12.5 J, determine,
   a. The compression of the spring
   b. The spring constant

7.5 Combination of springs

When several springs are combined together, their stiffness changes depending on how they are combined in the system. If the combination makes the springs stiffer, the effective spring constant is large and the total extension is less than the extension of a single spring.

Springs in Parallel

Fig. 7.7 shows a system of two springs connected in parallel. Spring 1 and 2 have spring constants k_1 and k_2 respectively.

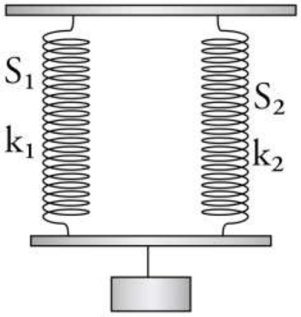


Fig. 7.7

In this combination, the system is stiffer than a single spring. The spring constant is thus the sum of individual spring constants.

$$k_{eq} = k_1 + k_2$$

If the springs are identical, then the total extension is half the extension of a single spring.

$$e_t = \frac{e}{2} = \frac{F}{2k_1}$$

Example 7.7

Determine total extension in the setup shown in Fig. 7.8. The springs are identical and the constant of proportionality (k) of each is 100 Nm^{-1}.

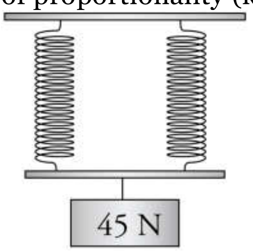


Fig. 7.8

Solution

$$k_{eq} = k_1 + k_2 = 100 + 100$$
$$= 200 Nm^{-1}$$
$$e = \frac{F}{k} = \frac{45}{200} = 0.225m$$

Alternatively;

$$e_t = \frac{e}{2} = \frac{1}{2}\left(\frac{45}{100}\right) = 0.225m$$

Springs in series

Fig. 7.9 shows two springs connected in series. When a constant force, F is applied on spring 2, the springs are extended and the total extension is the sum of extension of each spring.

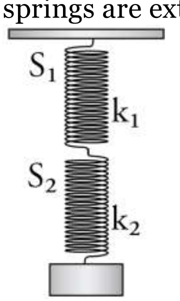


Fig. 7.9: Springs in series

$$\text{Total extension, } e = e_1 + e_2$$
$$= \frac{F}{k_1} + \frac{F}{k_2}$$
$$= F\left(\frac{1}{k_1} + \frac{1}{k_2}\right)$$

Therefore, for springs in series, effective spring constant $k_{eq,}$ is given by,

$$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} \ldots,$$

For two springs in series,

$$k_{eq} = \frac{k_1 \times k_2}{k_1 + k_2}$$

Example 7.8

It is observed that a 160 g mass produces an extension 4 cm when attached on a spring. Determine the total extension produced when three such springs are connected in series and parallel, as shown in the figure below.

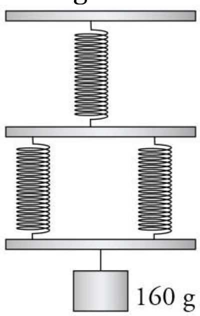


Fig. 7.10

Solution

Find the spring constant k

$F = 1.6\ N,\ e = 0.04\ m$

$$k = \frac{F}{e} = \frac{1.6}{0.04} = 40\ Nm^{-1}$$

Using extensions;

Extension of the top spring, e_1

$$e_1 = \frac{F}{k} = \frac{1.6}{40} = 0.04\ m$$

$$e_2 = \frac{e}{2} = \frac{0.04}{2} = 0.02\ m$$

$$e_t = e_1 + e_2$$
$$= 0.04 + 0.02 = 0.06\ m$$

Using spring constants

Spring constant for the two springs in parallel

$$k_1 = 2k = 80 Nm^{-1}$$

The equivalent spring pf the system in series

$$k_{eq} = \frac{k_1 \times k_2}{k_1 + k_2} = \frac{80 \times 40}{80 + 40} = \frac{80}{3} Nm^{-1}$$

$$e_t = \frac{F}{k_{eq}} = 1.6 \div \frac{80}{3}$$
$$= 1.6 \times \frac{3}{80} = 0.06\ m$$

Example 7.9

Given that the spiral springs in Fig. 7.11 below are identical with a spring constant of 200 N/m each, determine the total extension produced by the 300 N load.

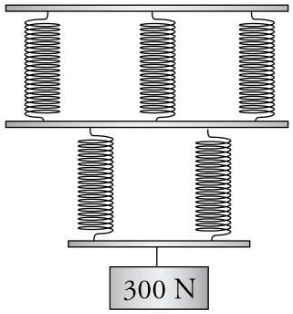
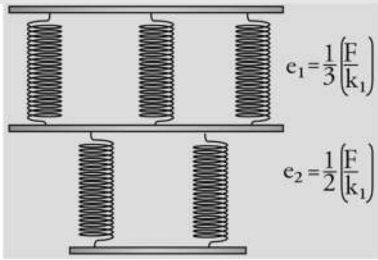


Fig. 7.11

Solution

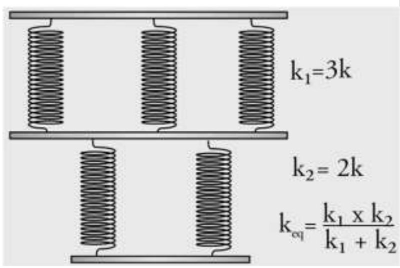


Using extensions,

$$e_1 = \frac{e}{3} = \frac{300}{3 \times 200} = 0.5\ m$$

$$e_2 = \frac{e}{2} = \frac{300}{2 \times 200} = 0.75\ m$$

$$\begin{aligned} e_t &= e_1 + e_2 \\ &= 0.5 + 0.75 \\ &= 1.25\ m \end{aligned}$$



Using spring constants

$$k_1 = 3k = 600 Nm^{-1}$$

$$k_2 = 2k = 400 Nm^{-1}$$

$$\begin{aligned} k_{eq} &= \frac{k_1 \times k_2}{k_1 + k_2} = \frac{600 \times 400}{600 + 400} \\ &= 240 Nm^{-1} \end{aligned}$$

$$e_t = \frac{F}{k_{eq}} = \frac{300}{240} = 1.25\ m$$

Test yourself 7.3

1. Two springs of spring constants 100 N/m and 200 N/m are connected in parallel. Determine the force that will produce an extension of 6 cm on these springs.
2. The figure below shows a spring system comprising identical springs. Given that the spring constant of each spring is 125 N/m, what is the extension produced when a mass of 2 kg N is attached on the system?

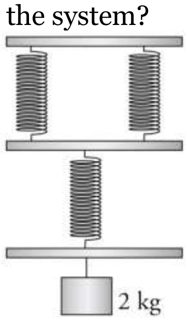


Fig. 7.12

7.6 Topical questions

1. State Hooke's Law
2. A shock-absorbing spring was seen to compress by a distance of 2 cm when a force of 500 N is exerted on the spring. What is the force constant k for this spring?
3. The diagram below shows a graph of force against extension for a certain spring.

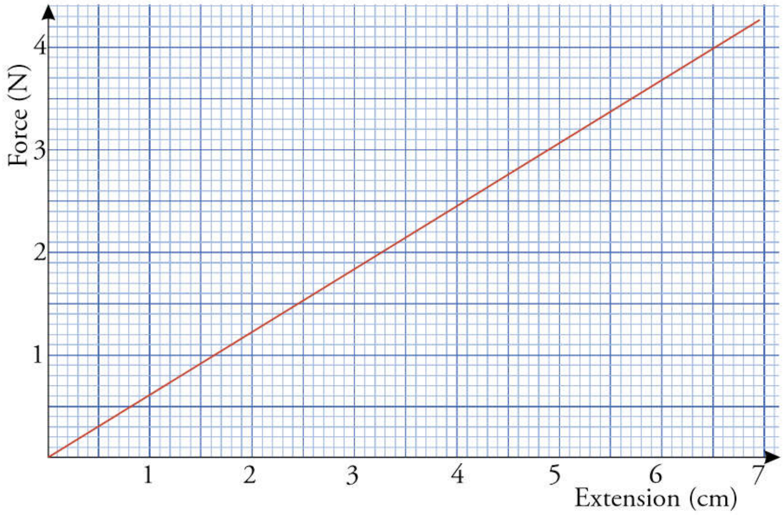


Fig. 7.13

   (a) What is the spring constant of the spring?
   (b) What force would cause two such springs placed in parallel to stretch by 10cm
   (c) State three factors that affect the proportionality constant of a helical spring.
4. Two identical springs of spring constant 120 Nm^{-1} each are connected in series. Find the effective spring constant.
5. A spring is stretched by 6 cm when 50 N force is loaded on it. If this force is increased to 180 N, how much would the spring stretch?
6. The figure below shows three identical springs of negligible weight. Two masses are attached as shown.

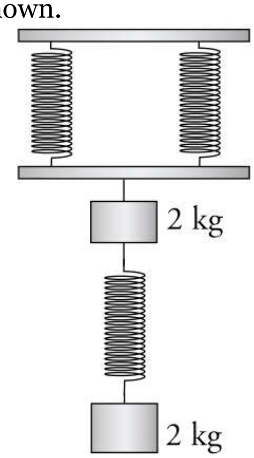


Fig. 7.14

   Determine the constant of each spring if the extension produced on the system is 10 cm.
7. The length of a spring when supporting a load of 10 N is found to be 13 cm. The same spring has a length of 17 cm when supporting a load of 25 N. What is the length of the spring when no load is supported?
8. A single light spring stretches by 4 cm when supporting a load of 3.6 kg. Five such springs are arranged and a 7.2 kg mass suspended on them as shown in the figure below.

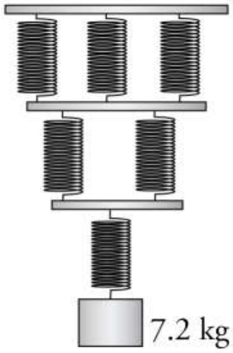
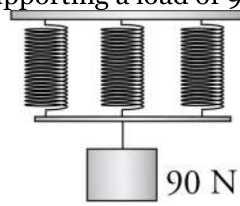
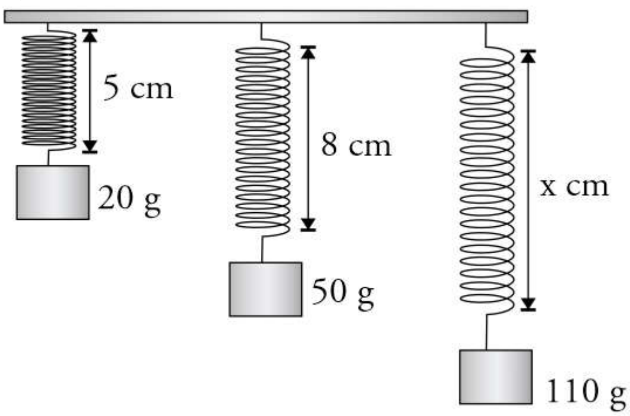


Fig. 7.15

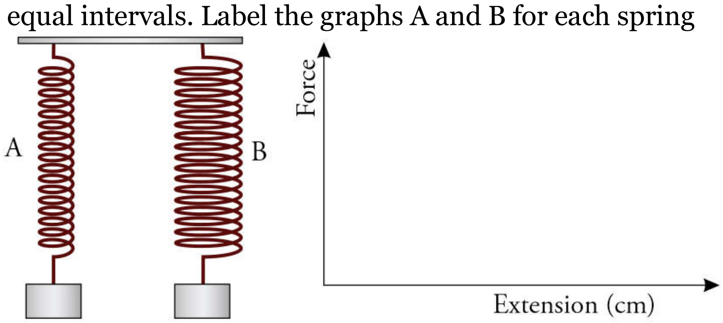
9. The figure below shows three identical springs with proportionality constant of 50 N/m supporting a load of 90 N



Calculate;
   (a) The extension in one spring
   (b) The extensive proportionality constant of the springs
10. The diagram below shows three identical springs which obey Hooke's law. Determine the length x.



11. Sketch a graph of length of a helical spring against compressing force until the coils of the spring are in contact
12. The figure below shows two springs made of the same material and same thickness. On the same graph, sketch a graph of force against extension when equal masses are loaded on the springs at equal intervals. Label the graphs A and B for each spring



13. A stretched spring supports a load of 20 N weight. When a load of 10 N is added, the spring stretches by an additional 6 cm. Determine the spring constant of the spring.